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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i)$ $a+bx_1+a+bx_2+a+bx_3+a+bx_4+a+bx_5$
   $= 5a+5(x_1+x_2+x_3+x_4+x_5)$

   b. $\sum_{y=0}^{5}f(x+y)$ $f(x)+f(x+1)+f(x+2)+f(x+3)+f(x+4)+f(x+5)$

   c. $\sum_{i=1}^{10}i^2$ $1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2$
   $= 1+4+9+16+25+36+49+64+81+100$
   $= 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$ $[2(1)+2]+[2(2)+3] = 4+4+3 = 11$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

   a. Find the value of b $0.5b+b+2.25b+2b+1.5b+0.5b+0.25b=1$
   $8b=1$
   Then $b=\frac{1}{8}=0.125$

   b. Find the answer for $P(X \le 2)$
   $P(X\le2) = P(X=2)+P(X=1)+P(X=0)+P(X=-1)+P(X=-2)$
   $= 1.5b+2b+2.25b+b+0.5b = 7.25b = 0.90625$
   ≈ 0.906

   c. Find the answer for $P(-2 \le X \le 3)$
   $P(-2\le X\le3) = P(X=-2)+P(X=-1)+P(X=0)+P(X=1)+P(X=2)+P(X=3)$
   $= 1.5b+2b+2.25b+b+0.5b+0.5b = 7.75b = 0.96875$
   ≈ 0.969

   d. Find the answer for $P(X \ge 1)$
   $P(X\ge1) = P(X=1)+P(X=2)+P(X=3)+P(X=4)$
   $= 2b+1.5b+0.5b+0.25b$
   $= 4.25b = 0.53125 \approx 0.531$

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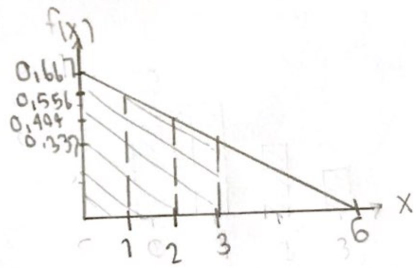
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3 \quad f(x)$$

a. Plot graph for $f(x)$



$P(X=0) = \frac{6}{9} = 0.667$

$P(X=1) = \frac{5}{9} = 0,556$

$P(X=2) = \frac{4}{9} = 0,444$

$P(X=3) = \frac{3}{9} = 0,333$

$2x\,dx = x^2$

$6x = 3x^2$

then $y=0 = -\frac{1}{9}x + \frac{6}{9}$

$x = \frac{6}{9}(9)$

$x = (6)$

$x=0$ then $y = \frac{6}{9} = \frac{2}{3}$

b. Find the answer for $P(1 \le X \le 3)$

$$F(x) = \int_1^3 f(x)\,d(x)$$

$$= \int_1^3 \left(-\frac{1}{9}x + \frac{6}{9}\right)dx$$

$$= -\frac{1}{18}x^2 + \frac{6}{9}x \Big|_1^3$$

$$= \left[-\frac{1}{18}(3)^2 + \frac{6}{9}(3)\right] - \left[-\frac{1}{18} + \frac{6}{9}\right]$$

$$= -\frac{1}{18}(9) + \frac{6}{9}(3) + \frac{1}{18} - \frac{6}{9}$$

$$= \frac{-8}{18} + \frac{6}{9}(2)$$

$$= \frac{16}{18} = 0,889$$

c. Find the answer for $P(X \ge 2)$

$$F(x) = \int_2^3 -\frac{1}{9}(x) + \frac{6}{9}$$

$$= -\frac{1}{18}x^2 + \frac{6}{9}x \Big|_2^3$$

$$= -\frac{9}{18} + \frac{18(2)}{18} + \frac{4}{18} - \frac{24}{18} = \frac{7}{18} = 0,389$$

d. Find the expected value of X

$$E(x) = \int_0^3 x f(x)\,dx$$

$$= \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right)(dx)$$

$$= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6x}{9}\right)dx$$

$$= -\frac{1}{27}x^3 + \frac{3}{9}x^2 \Big|_0^3$$

$$= -\frac{1}{27}(3)(3)(3) + \frac{3(9)}{9} = -1 + 3 = 2 \#$$

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4. Let random variable X be the outcome of throwing one dice (6 ด้าน) and random variable Y be the outcome of tossing one coin (2 ด้าน). Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

X \ Y	1	2	3	4	5	6	
0	$\frac{1}{6}\times\frac{1}{2} = \frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{6}{12}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{6}{12}$
	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{12}{12} = 1$

b

C

   b. Find the marginal probability distribution function (PDF) of X

$P(X=0) = \frac{6}{12}$

$P(X=1) = \frac{6}{12}$

   c. Find the marginal probability distribution function (PDF) of Y

$P(Y=1) = P(Y=2) = P(Y=3) = P(Y=4) = P(Y=5) = P(Y=6) = \frac{2}{12}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$P(X \mid Y=1) = \dfrac{P(X=x, Y=1)}{P(Y=1)} = \dfrac{\frac{2}{12}}{\frac{2}{12}} = 1$

   e. Find the expected value of X given Y is equal to 1

$E(X \mid Y=1) = \sum_i x_i P(X=x \mid Y=1) = 0+1 = 1$

   f. Find the variance of X given Y is equal to 1

$\mathrm{var}(X) = E(X^2) - (E(X))^2$

$= \sum_i x_i^2 P(X=x \mid Y=1) - 1^2$

$= 1^2(1) - 1 = 0 \rightarrow$ constant value no varies

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left(\frac{1}{3}(X_1 + X_2 + X_3)\right)$$
$$= \frac{1}{3}E(X_1 + X_2 + X_3)$$
$$= \frac{1}{3}(\mu_x + \mu_x + \mu_x)$$
$$= \frac{1}{3} 3\mu_x$$
$$= \mu_x$$

$$var(\bar{X}) = var\left(\frac{1}{3}(X_1 + X_2 + X_3)\right)$$
$$= \left(\frac{1}{3}\right)^2 [var(X_1) + var(X_2) + var(X_3) + cov(X_1, X_2) + cov(X_2, X_3) + cov(X_3, X_1)]$$
$$= \frac{1}{9}\left[3\sigma_x^2 + \frac{3}{4}\sigma_x^2\right]$$
$$= \frac{1}{\cancel{9}_3}\left[\frac{\cancel{15}^5}{4}\sigma_x^2\right]$$
$$= \frac{5}{12}(\sigma_x^2)$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

COV=0

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of u and σ

$$E(\bar{X}) = E\left(\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right)$$
$$= \frac{1}{4}\{E(X_1) + E(X_2) + E(X_3) + E(X_4)\}$$
$$= \frac{1}{4}\{\mu_x + \mu_x + \mu_x + \mu_x\}$$
$$= \frac{1}{4}(4)\mu_x$$
$$= \mu_x$$

independent → covariance is 0

$$var(\bar{X}) = var\left(\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right)$$
$$= \frac{1}{16}\{var(X_1) + var(X_2) + var(X_3) + var(X_4)\}$$
$$= \frac{1}{16}\{\sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \sigma_x^2\}$$
$$= \frac{1}{16} 4\sigma_x^2$$
$$= \frac{\sigma_x^2}{4} = 0.25\sigma_x^2$$

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b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$E(\tilde{X}) = E\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right)$$

$$= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4)$$

$$= \frac{1}{\cancel{8}4}\{\cancel{2}\mu_x\} + \frac{1}{4}\{\mu_x\} + \frac{1}{2}\mu_x$$

$$= \frac{1}{4}\mu_x + \frac{1}{4}\mu_x + \frac{1}{2}\mu_x$$

$$= \frac{1}{2}\mu_x + \frac{1}{2}\mu_x$$

$$= \mu_x \;\#$$

$\therefore \tilde{x}$ is unbiased estimator of μ

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$\mathrm{Var}(\tilde{x})$

$$= \frac{1}{64}\sigma_x^2 + \frac{1}{16}\sigma_x^2 + \frac{1}{64}\sigma_x^2 + \frac{1}{4}\sigma_x^2$$

$$= \frac{1}{32}\sigma_x^2 + \frac{1}{16}\sigma_x^2 + \frac{1}{4}\sigma_x^2$$

$$= \frac{1+2+8}{32}\sigma_x^2$$

$$= \frac{11}{32}\sigma_x^2 = 0.34375\,\sigma_x^2 \approx 0.344\,\sigma_x^2$$

$\bar{x}$ is better because the $\mathrm{var}(\bar{x}) \le \mathrm{var}(\tilde{x})$

Then $\bar{x}$ is more efficient estimator